



Mathematical Techniques Employed in Planning a Construction Project: Case Study on the Construction of Retaining Walls

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Abstract: The importance of planning and designing in construction projects is most apparent during the implementation of the respective activities; whereas the implications that normally arise from poorly controlled activities often result in overspending, delays, substandard products or complete failure of the project. There is the need for a comprehensive plan during the execution and control of construction activities; this is more so when the project is considered to be complicated by a variety of objectives. By employing mathematical techniques to the planning and designing of a project, it is theorized that this will enhance the administration of those matters influencing the project parameters of cost, time, quality and scope. The idea is applied to the study of a real project that involves the construction of retaining walls, so to justify the key steps in planning, moreover to integrate the parameters via mathematical techniques so to define the project baseline plan: *inferential statistics* applied in quality assurance, *linear programming* applied in the work breakdown structure, and *decision theory* applied in the networking/scheduling.

Keywords: Project Planning, Mathematical Technique, Project Management, Project Baseline Plan, Project Parameters

1. Introduction

The life cycle of the construction project is divided into four stages, which may be expressed as: feasibility, design/planning, construction, handover/defects liability period (Project Management Institute, (2008), Fourth edition, pp. 5, 6).

- I. The feasibility stage is first and includes an analysis of the possible alternative solutions to the subject matter.
- II. The stage of design and planning in this project encompasses surveys that are highly intellectual so to establish the baseline plan.
- III. The construction is the third stage and is the most intense as it relates to collective work of all type; it is the longest stage of this project life cycle and is generally where most of the resources are allotted. This third stage puts the project plans into action by executing procedures, schedules, and templates that were strategically developed in the previous stage. It is during this period that the objectives of the project are implemented/realized.
- IV. The defects liability period or handover are management processes that are undertaken in the last stage of the project lifecycle.

1.1. The project baseline plan

The project baseline plan must possess features that will allow the management body of the project:

- to make iterative planning and predictions;
- to assess alternative methods for achieving the objective;
- to compare project progression with contemporary plans and objectives;
- to obtain precise information on activities so to promote efficacy during project administration and implementation.

The task to manage any project can be quite cumbersome and even complicated; this is true for projects that have simple steps in their execution as well as those that require a high level of intricacy during their undertaking. The project baseline plan often determines the degree of management or type of intervention that is required during project

execution; it is to say that a detailed baseline plan will likely facilitate the administrative processes involved as well as support project success. On the contrary, the deficiencies in these plans can produce undesirable results and even cause disasters. At the stage of project implementation, it is important that this plan is not "in an open state" or susceptible to the diverse influences that surround the project.

Whereas most problems in project management arise from the interpretation of ambiguous information; can Mathematical techniques bring a clear definition to the project baseline plan? Will the scientific integration of project activities help in establishing the respective limits of, and the requirements for said activities?

The baseline plan must take into account four parameters; they are cost, time, quality and scope; this is considered critical in meeting the objective of the project. The parameters are co-dependent in that: an increased quality/scope generally implies increased cost and increased time. Meanwhile, a tight constraint on time signify reduced quality/scope and increased costs. Also a limited or fixed budget typically result in a reduced quality/scope and increased time. The mathematical analysis of the parameters involved in the project (to construct the retaining walls) aim to provide justifications for the project activities as well as the technical paths are chosen to achieve these activities, careful management of these four parameters should optimize the project outputs.

2. Methodology

The management process first includes establishing the project baseline plan; the second is to put this plan into action and achieve the objective. The key for effective control of a project is to periodically measure the real progression of activities and compare it with that of the baseline plan; this allows project managers to take quick corrective actions during the execution of works. Therefore, it is important to dedicate time in planning a project for this is critical to its management and success; in other words, the project baseline plan facilitates the processes of administration during the execution of the project objectives.

Fundamentally, the project baseline plan for the construction of the retaining walls consists of technically defining the four parameters (scope, time, cost and quality) and their degree of involvement in the activities of works necessary to accomplish the respective output; as such there are three tasks deemed necessary for defining these parameters. They are as follows:

- Quality assuring
- Work structuring
- Networking/Scheduling

Also, the idea entails establishing the project baseline plan for the construction by employing mathematical techniques. *Inferential Statistics* to achieve the standard for quality of raw materials required by the construction - quality assuring; *Linear Programming* to define in a systematic manner, the cost and scope of project activities- work structuring; *Decision Theory* to provide the logistics in scheduling the project activities - networking/scheduling.

2.1 Quality assuring

Before one can begin to build the retaining wall, it is important to sort out the raw materials. These materials include boulder, gravel, sand and cement. It is a criterion to verify the compressive strength of the concrete during the project execution; this is done by taking samples during the construction stage for analysis at laboratories. It is also necessary to justify the physical properties of the boulder, gravel, and sand; however, it would prove costly, tedious and a waste of time to measure the compressive strength of each boulder (especially if the procedure utilized for achieving the measurement result in the destruction of the item). The most logical approach would be to investigate notable sites of geological deposits of the raw materials, henceforth to apply scientific procedures for sampling the boulders and derive at a sample distribution. Further, mathematical analysis can then be used to estimate the characteristics of the physical nature of the respective population of that material. This should be done for all three raw materials (boulder, gravel, sand) that constitute the retaining wall structure.

Statistical techniques have proven to be effective tools that can be used to analyze and process raw data. This science manipulates the numerical properties of measurements that are derived from the raw data to make inferences. The procedures of statistical inferences begin with the analysis of one or more empirical distributions to derive at a

theoretical probability distribution. The specific point at which these theoretical distributions are applicable is centered in the similarities that exist between the samples and the populations.

The mathematical operations that constitute inferential statistics can readily deduce the physical nature of the raw materials. The properties of measurements gathered from a particular sample (for example, the compressive strengths of boulders in mega Pascal) are the main elements to form the basis for the logical inferences that can be made. Note that these deductions are dependent on the conditions of the study, in other words, the method of the sample selection. The central issues in sample selection are:

- The sample shall represent the entire population;
- the sample shall be unbiased whereas all the elements of the population has an equal probability of being selected for the sample;
- the sample size shall be sufficient to logically deduce the nature of the population within the required levels of confidence.

2.2 Work structuring

This is predominantly known as the Work Breakdown Structure (WBS). The WBS typically orientates project components into groups so to organize and define all the activities of the project. In other words, project deliverables are divided into manageable components and subsequently defined so to facilitate other project activities such as planning, construction, and handover. In general, there are several tasks involved to achieve the structural breakdown of the construction successfully works for the retaining walls; these tasks are expressed in the following proceedings:

- Identifying the activities. To give clarity on the plan's workload. The activities should be subdivided and detailed such that the time and the scope for each deliverable can be effectively estimated.
- Organizing the activities. The relation between each activity are determined whereby the dependencies between them allows for a logical sequence; that is, which activity must be completed before the start-up of another activity.
- Duration of the activities. Specialists or experienced personnel analyze and quantify the duration of the respective activities.
- Integration of the activities. Each activity is synchronized/coordinated within a working structure according to its start and completion date. Also the personnel responsible for its implementation may be asserted.

2.3 Networking/Scheduling

The construction project constitutes of a set of activities to be performed in a specific sequence to completion. The said activities are interrelated by a logical sequence in the sense that they are all linked by their starting and or finishing. Each construction activity requires time and resource for its execution. The objective of networking is to minimize the total project time subject to the constraints of the project resources; this involves the "Theory of Decisions" where the calculations established by the "Critical Path Method (CPM)" are employed. Note that, the duration of activity in the CPM are deduced from historical events or experience.

The network diagram represents the construction works in that it shows the precedence relationships of the respective activities and their times. The diagram applies the use of "arrows" to represent activities that will consume resource and time. The antecedence of the activities is recognized with the use of "nodes" to represent events; the latter do not consume any resources. Events merely indicate the points in time where activities are initiated and/or terminated; they can be considered as points for specific deliverables throughout the execution or construction stage of the project.

Through computations performed on the network (arrow) diagram (chart), the following information can be inferred via CPM:

- I. Start and completion times for each event
- II. Critical and noncritical activities
- III. Total float and free float times for activities.

3. Results

The results should provide the feasibility of achieving the project quality/scope over a proposed period at a respective cost.

3.1 Quality assurance

It is important to note that, the boulders are first in priority of the raw materials and must possess a compressive strength of at least 20 mPa.

Listed below are the magnitudes of the compressive strengths for a sample of 30 boulders, each measured to the nearest "mega Pascal". The mean of the sample is calculated, as well as the sum of squared deviates, the variance, and the standard deviation.

Table 1: Magnitudes of the compressive strengths

19	19	20	20	20	$M_x = 23$
21	21	21	22	22	$SS = 121$
22	22	22	23	23	$s^2 = 4.16$
23	23	23	23	24	$s = \pm 2.04$
24	24	24	25	25	
25	25	26	26	26	

Because the variability of the population being studied is derived only via estimation, the sampling distribution of t is used as the respective reference. This corresponds to the t -distribution pertaining to $df=29$.

For a sample distribution of t for $df=29$, 95% of the distribution is within ± 2.05 units of t with regards to the mean; whereas each unit of t has an equivalence of 0.37 units on the respective scale of measurement. For a confidence level of 95%, the estimated mean of the source population of boulders is 23 ± 0.76 mega Pascal.

3.2 Work breakdown structure

Table 2: Work breakdown structure

Item	Description of activity	Unit	Quantity	Immediate predecessor activities	Duration (in weeks)
1	Operational/Site Activation				
1.1	Data gathering			–	–
1.2	Mobilization of equipment.			1.1	1
1.3	Production of raw materials			1.1	4
	Boulder	m ³	4050		
	Gravel	m ³	1800		
	Sand	m ³	457		
2	Retaining Wall Construction				
2.1	Excavation	m ³	1000	1.2	2
2.2	Rubble masonry	m ³	3860	2.1, 1.3	28
	Backfilling	m ³	1230		
2.3	Parapet	m	70	3.1	2
3	Drainage				
3.1	Spillway	m ²	35	3.2	2
3.2	Soakaway	m ³	27	2.2	2
3.3	Asphaltic concrete cross drain	m ²	21	4.4	1
3.4	Concrete V-drain	m	60	4.3	2
4	Road Surfacing				
4.1	Prepare earth formation	m ²	360	2.2, 3.2, 3.1, 2.3	1
4.2	Prepare sub-base material - river shingle or crushed aggregate	m ³	108	4.1	1
4.3	Prepare base material - marl	m ³	72	4.2	4
4.4	Asphaltic Concrete Pavement	m ²	450	3.4	1

3.3 Networking/Scheduling

Suppose in assuring the quality of the construction, the investigations done via inferential statistics (section 2.1) have identified and certified three mining sites for the production of the raw materials (boulders, gravel, sand); also, it is a project criterion to utilize all of the three mines for a period no less than six months. Furthermore, each mine is characterized by its own processes, machinery, and operational cost.

The maximum production capacity of each mine for a period of six months is expressed as follows:

Mine	Boulder (m ³)	Gravel (m ³)	Sand (m ³)
1	2500	1000	300
2	1800	1000	300
3	1300	1900	200

Whereas the cost (per m³) of material in each one of the mines is as follows:

Mine	Boulder (\$/m ³)	Gravel (\$/m ³)	Sand (\$/m ³)
1	30	45	35
2	35	40	40
3	40	30	50

Mine 1 has the machinery to produce the three raw materials, one at a time, and a maximum production time of 705 hours which must be used in its entirety. The consumption of time in hours for the production of a cubic meter of a boulder and a cubic meter of gravel is of 0.2 hours, 0.4 hours, and in an hour, it is capable of producing 1 m³ of sand. To produce gravel and sand regardless of the amount implies an additional expenditure of \$200 and \$100 and consumption in a time of 2 hours and 0.5 hours respectively.

Mine 2 machinery has a maximum production time of 480 hours, the consumption of time per m³ of each material is 0.25, 0.45, 1.5 hours for a boulder, gravel, and sand respectively. To produce gravel and sand regardless of the amount, will cost an additional \$150 and \$80 as well as consume times of 2 and 1 hours respectively.

The processes of Mine 3 are more advanced than that of the other two mines and can produce the three materials but is designed primarily for the production of gravel. Its processes consume 2, 0.23 and 1.6 hours in the production of 1m³ of stone, 1m³ gravel, and 1m³ sand. The maximum production time of its machinery is 750 hours.

Mathematical models can be used to analyze the volumes of production of each material at the mines, taking into account all the constraints that exist in their operations. This linear programming integrates the operations in all three mines and determines the specific quantity of each material be taken from each mine with the principal objective of lowering project costs. The approach helps to minimize the cost of the raw materials henceforth the cost of the project; the models also provide a scientific justification in the procurement of these raw materials with regards to each mine. With the help of Microsoft Excel (Solver), the quantities of each raw material in cubic meters that are to be produced by each mine are shown below tabulated.

Mine	Boulder (m ³)	Gravel (m ³)	Sand (m ³)
1	2500		204
2	1550		61
3		1800	192

The model illustrates the production of raw materials at a minimum total cost of 202,610 dollars [(2500*30) + (204*35) + (1550*35) + (61*40) + (1800*30) + (192*50) + 100+80 = 202,610].

Note that further mathematical computations can be used so to plan the transportation of these materials and in so doing optimize the associated cost.

Table 3: Networking via CPM

Activity	Duration (weeks)	Latest finish time (weeks)	Earliest start time (weeks)	Float (weeks)
1.1 - 1.2	1	2	0	1
1.1 - 1.3	4	4	0	0
1.2 - 2.1	2	4	1	1
2.1 - 2.2	28	32	3	1
1.3 - 2.2	28	32	4	0
2.2 - 3.2	2	34	32	0
3.2 - 4.1	1	39	34	4
2.2 - 4.1	1	39	32	6
3.1 - 4.1	1	39	36	2
3.2 - 3.1	2	36	34	0
3.1 - 2.3	2	38	36	0
2.3 - 4.1	1	39	38	0
4.1 - 4.2	1	40	39	0
4.2 - 4.3	4	44	40	0
4.3 - 3.4	2	46	44	0
3.4 - 4.4	1	47	47	0

Note that the critical route is characterized by a float of zero.

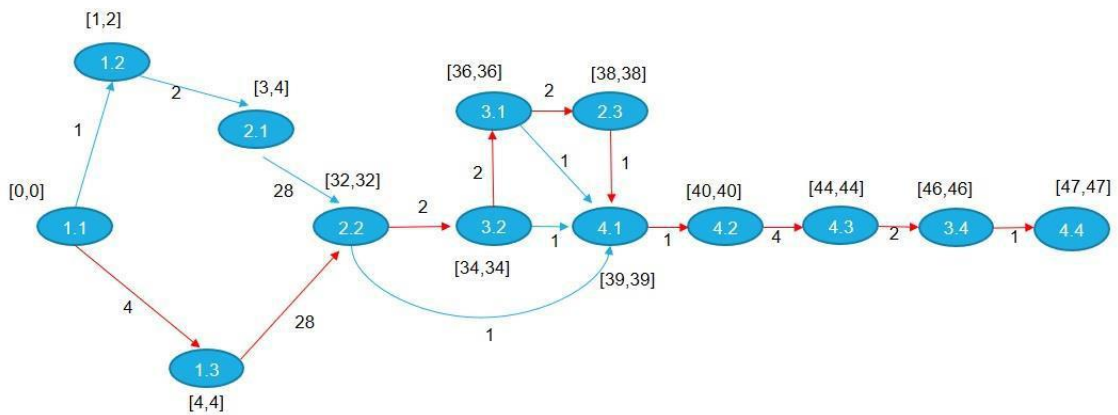


Figure 1: Network Diagram via CPM

4. Discussion

The idea to establish the project baseline plan should ultimately facilitate the management of the project. A baseline plan that is rooted in the principles of mathematical science is in itself strategic; this is more obvious for projects that are complicated with multiple objectives and/or subjective influences. Mathematical techniques can bring to light the project objectives in a manner that is precise and clear. The techniques employed by the project baseline plan should provide tangible evidence, whereas to justify the parameters of the construction: scope, quality, cost and time. The mathematical concepts employed, promotes a systematic evaluation of the project parameters and aids in the decision-making processes before and during the execution of works.

The application of inferential statistics is necessary for establishing the sites for mining the raw materials that will be utilized in the construction; whereby the application certifies the quality of the raw materials to be used in the construction. Through research of the project operations, the mathematical models employed will indeed provide a scientific integration of the project activities and therefore elucidate the project path.

5. Conclusion

Mathematical procedures can provide the framework for planning the construction project. The scientific approach of employing mathematical principles in the project baseline plan will:

- a) Safeguard the natural and built environment that is associated with the construction;
- b) Ensure that basic objectives of economy, efficiency and efficacy are met;
- c) Assist in achieving the project deliverables within the estimated time, cost, quality and scope;
- d) Control and guide the management body in making technical decisions;
- e) Bring precision to the technical specifications that are integral to project activities;
- f) Tend to eliminate subjective influences involved in project activities.

The processes of administration and management are much easier with the aid of graphic designs of the project activities; furthermore, the calculations accomplished via the techniques of decision theory, linear programming, and inferential statistics; provide a scientific integration of the project activities and elucidate the project path. An adequate project plan can reduce the respective costs including implementation time as well as aid in achieving the scope of works regarding the quality and the quantity of raw materials required for each activity.

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