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Multi-Product Production Optimization of Maintenance Integrated into the Control Chart Under Service Level and Quality Constraints

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Abstract: This work proposes a combined production, maintenance, and quality control for a supply chain management system. In this work, we consider a joint model production system consisting of a single machine producing multi-products connected to a multi-warehouse to satisfy changing and characterized customer demand for all the products throughout the production horizon. The work will answer some managerial questions about which product to produce and how much each is to make. Then the need for a maintenance strategy corresponding to the optimal production plan. The unreliable production system is prone to random failure, directly impacting product quality. The variance in production rate, inventory management, and utilization affects the status of the production system and its maintenance evolution. The control of the process and its quality is carried out using a multi-level statistical process control (SPC) and its tool, "control chart of average." The goal is to create a constraint optimization model that reduces total cost while meeting changing customer requests across many warehouses and over time. We consider the impact of production rate on the production machine degradation, the increasing failure model, and maintenance methods based on the optimal parameters of the control chart.

Keywords: Varying demand, Production optimization, Control chart, Maintenance strategy, Cost minimization

1. Introduction

These days, with the changes in worldwide economic conditions, request high-quality products at a low cost. Characterized production planning coupled with corresponding maintenance tactics based on statistical control chart of quality will be an essential aspect of industrial engineering management. To satisfy varying demands, the need to use a single machine to produce different products in many enterprises, such as cosmetic and pharmaceutical companies. The creasing cost of production, product holding, and delivery challenges have forced more constraints on producers to stay competitive, Reference (Abubakar et al., 2020). Moreover, customers are becoming more urgent, and their multiple needs in terms of consciousness and demand (Hajej et al., 2021). At the same time, production frameworks are getting more complex and prone to numerous instabilities (Ben-Daya and Rahim, 2000). Therefore, there is a need to create modern industrial strategies of more structured and integrated production planning, maintenance, and quality optimization tailored to reduce costs and satisfy customers' different necessities (Megoze Pongha et al., 2022). The production planning optimization of the maintenance technique has continuously been an incredible challenge for industrial companies (Rivera-Gómez et al., 2021). In any case, the item or benefit quality diminishes when it influences either the method that made it by expanding costs, time, and imperatives or controlling the ultimate item or benefit utilized by the client (Aarab et al., 2017).

All aspects of production, inventory, delivery, maintenance, and customer requirements are affected by quality and quality management (cost and service satisfaction). Several process improvement strategies, such as Six Sigma Methods, have been investigated in the literature to achieve similarity of quality and fulfillment of customer needs in the manufacturing process—analysis of the causes and implications of the system measurement. The Control chart, which is based on the statistical process control (SPC) methodology, is one of the fundamental tools used by Six Sigma to enhance the quality of goods and processes by minimizing process variance and the accompanying high defect rate (Antosz and Stadnicka, 2018; Sharikh et al., 2019; Sinha and Singh, n.d.). This research focuses on the Six Sigma approach, which can be used in manufacturing or service industries.

On the other hand, among the first acts of a hierarchical production process decision-making action—the failure or dysfunction of the production system considered—are optimal maintenance and production plans that minimize overall cost, including production, inventory, delivery, and maintenance. As a result, the integration of statistical process control, maintenance, and production is seen as a solution to the cost and no conformal-output problems that plague production systems. As a result, several researchers have looked into integrating the control chart "x-bar" and the periodic preventive maintenance approach to stabilize the process and avoid the creation of non-conformity. In their work "Effect of maintenance on the economic design of x-control chart," reference (Ben-Daya and Makhdoum, 1998) investigated the impact of maintenance on cost; however, they ignored stock deficiency. Moreover, its extension by (Rasay et al., 2022; Si et al., 2018) suggested a reliability and maintenance structure framework for a two-state process optimizing decision variables of a manufacturing system to determine discrete preventive maintenance times. The optimal production plan and maintenance strategy required the company to manufacture products that satisfy a varying demands over future periods. In this context, Ref. (Hadian et al., 2021a) studied the problem of integrated maintenance to production strategies by analyzing the role of buffer stocks in increasing system productivity. Ref. (Sadok et al., 2012) determined the optimal production and maintenance plans simultaneously, and the optimal delivery plan considered the delivery time, machine failures, random demand, and withdrawal rights. Indeed, delivery time and transported quantity are essential characteristics of manufacturing systems. Thus many manufacturers are working to reduce transportation delays such as the delivery time, which is the period that the part takes between a manufacturing store and a purchase warehouse (customer), which usually significantly impacts performance measures. Ref. (Lee, 2005) developed a model for supporting investment strategies about inventory and preventive maintenance in an imperfect production system that considers the delivery time to the customer.

Various authors looked into multiple aspects of production and upkeep. The company's ideal production plan and maintenance strategy needed them to develop items to meet changing demand. In this regard, in their paper "Automatic transfer lines with buffer stock," reference (Buzacott, 1967) was one of the first to improve productivity. They studied the impact of buffer stocks in boosting system productivity to consider integrated maintenance to production plans. (Sett et al., 2017) Ref. Researchers used a multi-objective optimization inventory model to establish the appropriate just-in-time buffer required for efficient operational management. The failure rate rises with time, and the use of the machine, but the latter is unusual. Most researchers, once again, presume a flawless production system. Most studies in the literature, we found, provided for the coordination of two essential capacities: production, maintenance, and quality. Production and maintenance integration, followed by production with quality or maintenance with quality, and most recently, the complete trends are trying to address all three elements. The originality of this work is that it uses the results of the control chart parameters and the correlation between production and maintenance to determine the best presentation of the combination of production rate, delivery quantity plans, and maintenance tactics according to customer service level requirements. Ref. (Zied et al., 2014) decided on the ideal production, delivery, and maintenance accounting conveyance time, machine disappointment, and random demand in addition to the withdrawal right in their work "Impact of delivery time on Optimal Production, Delivery, and Maintenance Planning." Manufacturing systems must, after all, have a delivery time and a carried amount. Production systems must, after all, have a delivery time, quantity, and acceptable cost. As a result, many companies are attempting to eliminate transportation delays such as delivery time, when it takes products to travel from a production store to a customer's warehouse, and substantially impacts performance metrics. Ref. (Cheng and Li, 2020) established an integrated model for a flawed production system that dealt with lot sizing, quality, and joint and condition-based maintenance. Ref. (Cheng et al., 2018) pandered with an imperfect production system that used inferior rework processes and imperfect preventive maintenance to determine the ideal economic production quantity (EPQ), multiple inspections, and inspection times. Ref. (Nahas, 2017) constructed a model of a serial production line consisting of several faulty machines to several buffers to determine the appropriate Preventive maintenance policy and stock size. Ref. (Dellagi et al., 2017) discovered that production rate fluctuations between periods significantly impact production maintenance plans and total incurred costs, allowing for the optimal production and preventive maintenance plan to be determined while considering constraints related to production system capacity. Reference (Baklouti et al., 2020) integrated the model of production quality. This research looked at a randomly failing solar power system supposed to serve random demand while maintaining a certain service level, hence overcoming production quality difficulties. To lower the failure rate of non-conformal products and increase impacts on performance measures, we may extend this approach to account for other critical manufacturing factors such as delivery quantity, cost, time, and quality control. Other researchers focused on upkeep and quality. Ref. (Pandey et al., 2011) developed a methodology for joint optimization of maintenance planning, process quality, and production schedules but did not consider the effects of inventory control shortages. They discovered that production rate fluctuations between periods significantly impact production maintenance plans and total expenses incurred, enabling the determination of the best production and preventive maintenance plan while considering production system capacity restrictions. Ref. (Salmasnia et al., 2017) developed a collaborative design of production run length, maintenance policy, and control chart with many assignable reasons. (Hadian et al., 2021b) investigated the integration of Production Maintenance and Quality for the collective determination of Economic Production Quantity and the periodic Preventive Maintenance level for an imperfect system that switches to a control state after a random time interval. The study showed how Preventive Maintenance reduces non-quality. The goal of Ref. (Dutoit et al., 2019) was to detect process shifts and subject the production system to them—production Process Simulation at the point of failure, prone to maintenance and quality control. Moreover, quality base maintenance, thereby resolving the issue. Recently, (Gölbaşı and Demirel, 2017) implemented an integrated production scheduling and maintenance planning model that uses a simulation algorithm. Develop an analytical model that integrates production sampling quality control and maintenance of a critical production system with an AOQL constraint in reference (Bouslah et al., 2016). However, the model assumed that a single quality attribute deteriorated with aging. Furthermore, products and processes are growing increasingly complex due to various causes, necessitating more quality checks and multiple inspections. Ref. (uit het Broek et al., 2021) worked on combining production, quality control, and condition-based maintenance for the flawed production system. They examine manufacturing with lot sizing and quality control based on inspection policies to determine the percentage of defectives.

Integrated production, maintenance, and supply chain management control chart under quality constraints in Reference (Lesage and Dehombreux, n.d.) to reduce the total cost conjugated manufacturing, delivery, and maintenance optimization were devised. In "A cost/benefit model for investments in inventory and preventive maintenance of an imperfect production system," Ref. (Rezg et al., 2004) developed a model for supporting inventory and preventive maintenance investment strategies in an imperfect production system while considering customer delivery time. Reference (Radhoui et al., 2010) created an integrated approach to production inventory control based on the age Preventive Maintenance strategy. A corrective maintenance action is prepared to return the machine to its original condition (AGAN). The work allows the ideal buffer stock size to be determined based on machine age and maintenance requirements. Ref. (Al-Salamah, 2018) proposes an integrated preventive maintenance quality model for an imperfect manufacturing system that includes two decision variables: buffer stock quantity and the rate of non-conform units for which one must take preventive maintenance actions. Reference (Cassady and Kutanoglu, 2005) created a model for optimizing production size and the time to perform Preventive Maintenance to reduce non-conform units when a process swings out of control. According to Ref. (Abubakar et al., 2022), production rate fluctuations between periods significantly impact production maintenance plans and overall incurred expenses, enabling the ideal production and preventive maintenance plan to be created while considering production system capacity.

Following the contributions in the existing literature, we identified some knowledge gaps. This study focuses on the statistical quality control integrated into maintenance and production plans to improve process reliability significantly. Also, reduce non-conformal losses and satisfy customers' varied requirements by improving quality and minimizing the total cost. The total price will include the total production of the multi-products, the entire inventory holding at the principal and multi-warehouse, and the comprehensive maintenance, which consists of the PM, CM, sampling inspection, and the non-quality product cost. Integrating the control chart on the optimal production plan and maintenance strategy required the company to manufacture products that satisfy a varying characterized demand over the entire period. In this context, the first actions of a hierarchical decision-making process - control of production rate and duration - the failure or dysfunction depends on the production system and is considered among the causes of non-conforming items. Hence, the solution to reducing cost is the integration of statistical process control on maintenance strategy, collaboration with production, and correlation of the customer service level, different product production, and quality bounds. A mathematical model will help determine the decision variables, minimizing the total production, maintenance, inventory, and quality developed. The remainder of the paper tests the solution's effectiveness using a numerical experiment to validate the solution's robustness, from which results are obtained and a conclusion drawn.

2. Production and Maintenance Problem

This study contributes to optimizing a production system composed of a single machine that produces two products (P1 and P2) to be stocked first in a primary production store. Then appropriate delivery quantities ($\theta_{i,1}, \theta_{i,2}, \dots \theta_{i,L}$) will be delivered to multi-customer ($d_{i,1}, d_{i,2}, \dots d_{i,L}$) purchase ware-houses ($w_1, w_2, \dots w_L$) with a delivery time τ_i ($i: 1, \dots L$). The system operates at a given service level θ_i, under the following constraints; $Prob[w_i(k) \geq 0] \geq \theta_i$, Production bounds: ${u^1}_{min} \leq u_1(k) \leq {u^1}_{max}$ for the product 1 and ${u^2}_{min} \leq u_2(k) \leq {u^2}_{max}$ for the product 2, over a finite horizon H, as shown in Figure 1 below. The production machine is subject to random breakdowns and repairs.

Production rates and inventory plans influence the deterioration of the machine. Consequently, the failure rate $\lambda(t)$ is not constant; it increases with time and variable production rate $u(k)$, affecting the reliability and capability of the production process responsible for non-conforming units. The goal is to control quality and machine failure rates by designing new integrated maintenance to production and delivery strategy under quality constraints by optimizing production and maintenance while considering the effects of production, stock, quality, and maintenance.

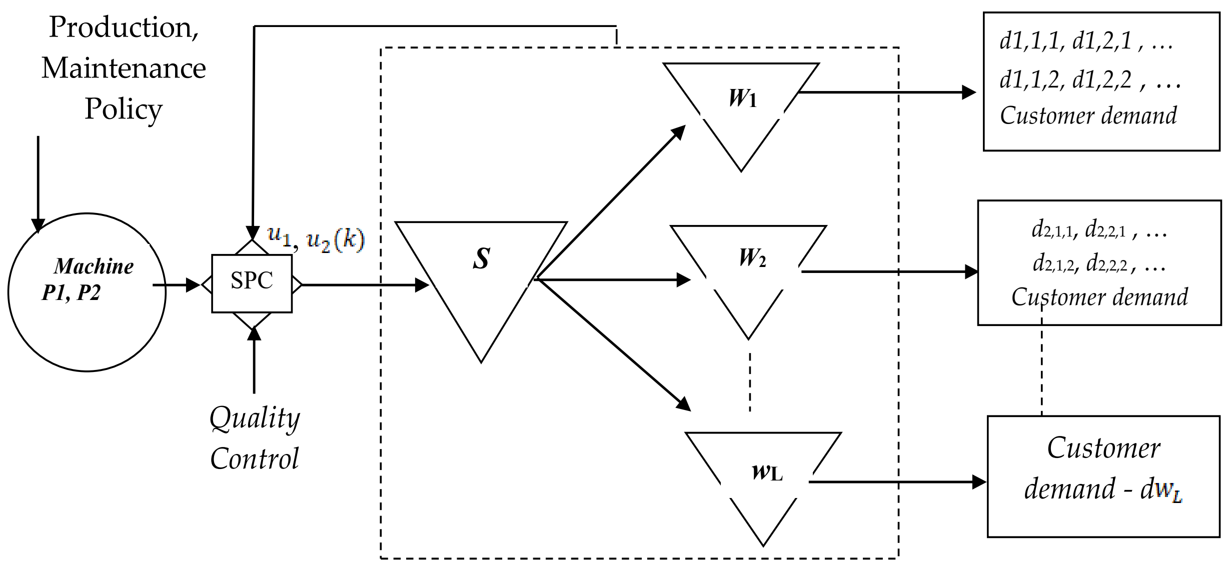

Figure 1. Production System, control chart, and Supply chain management

2.1 Notations

The following parameters are used in the mathematical formulation of the model:

$u_1(k)$:Production rate of product 1 during period k, $(k = 1, \dots, H)$ on machine M
$u_2(k)$:Production rate of product 2 during period k, $(k = 1, \dots, H)$ on machine M
$u^1{}_{max}$:maximal production rate of product 1 on machine M
$u^2{}_{max}$:maximal production rate of product 2 on machine M
$u^1{}_{min}$:minimum production rate of product 1 on machine M
$u^2{}_{min}$:minimum production rate of product 2 on machine M
$S(k)$:Inventory level of S at the end of period k, $(k = 0, 1, \dots, H-1)$
S :Principal Store S (Manufacturing Stock).
$w_{1,i}(k)$:Inventory level of the product 1 for each warehouse w_i and for period k, $(k = 0, 1, \dots, H-1)$
$w_{2,i}(k)$:Inventory level of the product 2 for each warehouse w_i and for period k, $(k = 0, 1, \dots, H-1)$
τ_i :Delivery time for all the products to all the warehouse w_i
$Q_{1,i}(k)$:Delivery rate of the product 1, for each warehouse w_i during period k, $(k = 0, 1, \dots, H-1)$
$Q_{2,i}(k)$:Delivery rate of the product 2, for each warehouse w_i during period k, $(k = 0, 1, \dots, H-1)$
L :Number of ware-houses
Δt :Length of a production period
$d_{1,i}(k)$:Average demand of the product 1, for each customer, and during period k, $(k: 1, 2, \dots, H)$
$d_{2,i}(k)$:Average demand of the product 2, for each customer, and during period k, $(k: 1, 2, \dots, H)$
$V_{di}(k)$:Variance of demand during period k, $(k: 0, 1, \dots, H)$ for each customer
H :Number of production periods in the planning horizon
$H.\Delta t$:Length of the finite planning horizon
c_{p1} :Unit production cost of product 1 on the machine M
c_{p2} :Unit production cost of product 2 on the machine M
c_{hs} :Inventory holding cost of unit product during one period at the principal store
c_{hi} : Inventory holding cost of unit product during one period at the ware-house w_i, $(i = 0, .. L)$ all product.
c_i :Unit cost of inspection

c_r :Unit cost of one defective unit
ACQ :Average cost of quality
c_{si} :Cost of Sampling inspection
c_{NCr} :Cost of non-Conformal items
θ_i : Probability index related to each customer I service level satisfaction.
j :Average number of samples to detect the 'out of control' state.
A_1 : Number of standard deviations between the center line of the control chart and the control limits P1.
A_2 : Number of standard deviations between the center line of the control chart and the control limits P2.
h :Sampling interval
c_1 :The magnitude of the shift to the 'out of control' state compared to the centerline in the case of P1
c_2 :The magnitude of the shift to the 'out of control' state compared to the centerline in the case of P2
$P(S_{1,1})$:The probability of non-detection of the shift to the control limits for product 1 sample
$P(S_{1,2})$:The probability of non-detection of the shift to the control limits for product 2 sample
$P(S_{2,1})$:The probability to trigger a false alarm for product 1 sample
$P(S_{2,2})$:The probability to trigger a false alarm for product 2 sample
C_{pm} :Cost of preventive maintenance action
C_{cm} : Cost of corrective maintenance action
C_M : Total maintenance cost
mu : Monetary unit.

2.2 Description of the Problem

This study presents the joint optimization of production and maintenance planning of the quality control problem. We consider a production system subject to random failure. The production rates influence the degradation degree of the machine. Consequently, the failure rate $\lambda(t)$ increases with time and production rate $u(k)$ and affects the production process's reliability, producing non-conforming products. The production machine is controlled at every h time interval of production period Δt by a quantitative quality statistic X with n measurements for the sample t. It is assumed that the individual measures of all the products ($P1$ & $P2$) must be between the upper control limit product 1 ($UCL1$) and the lower control limit product 1 ($LCL1$).

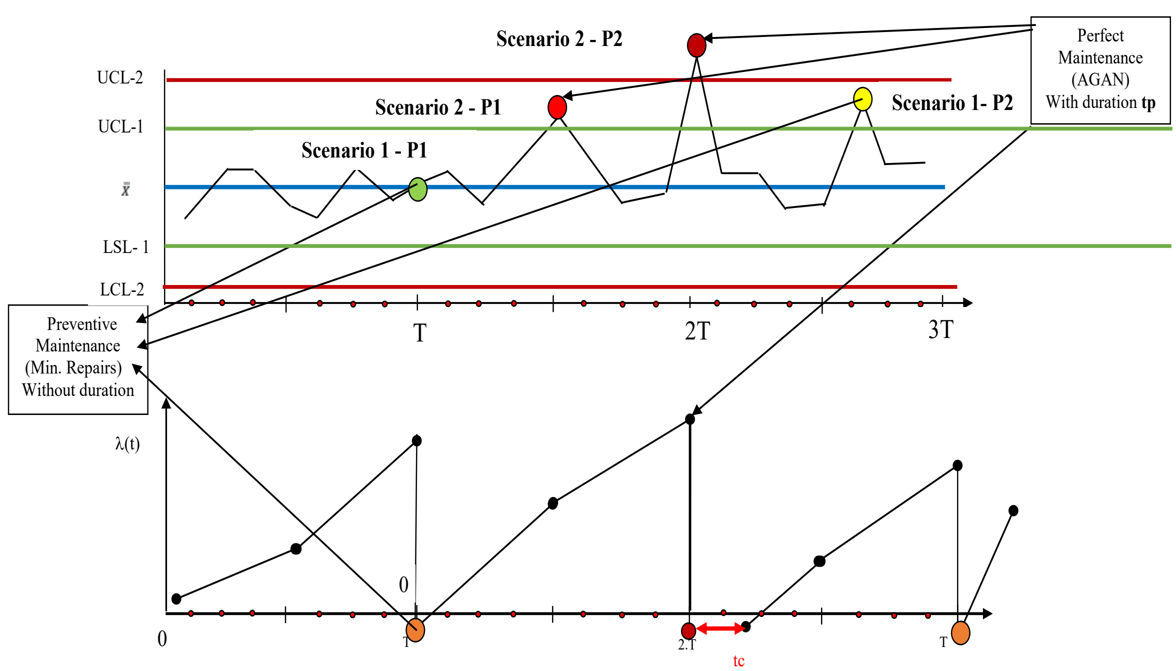

Figure 2. Integrated quality control and maintenance strategy

Similarly, product 2 must be between the upper control limit product 2 ($UCL2$) and the lower control limit product 2

($LCL2$). With a risk of the first species, the measurement results are saved in a measurement card. Moreover, assuming that the production process is stable when the Gaussian law of X_t is constant with a mean (μ_0) value and a standard deviation (σ_0) either known or well estimated for specific products. The process is under control when all statistics are found within the control limits: between the upper and the lower control limits. From the control chart, there are four possible outcomes, two of which occur in the control situation and the other two in the out-of-control state. According to Figure 2 below, the control states for the first and second products (P1 and P2) are situations I - 1 and I - 2, accordingly, while the out-of-control states for those products are scenarios II - 1 and II - 2.

The control limit for product 1 is ($LCL^1 \geq \bar{X}_t \geq UCL^1$), and product 2 is ($LCL^2 \geq \bar{X}_t \geq UCL^2$).

Where, $LCL^1 = \mu_{1,0} - \frac{A_1}{\sqrt{n_1}} \times \sigma_1$ (1)

And, $UCL^1 = \mu_{1,0} + \frac{A_1}{\sqrt{n_1}} \times \sigma_1$ (2)

Similarly, $LCL^2 = \mu_{2,0} - \frac{A_2}{\sqrt{n_2}} \times \sigma_2$ (3)

And, $UCL^2 = \mu_{2,0} + \frac{A_2}{\sqrt{n_2}} \times \sigma_2$ (4)

The mean value of the quality attributes can change from $\mu_0\ to\ \mu$. The production has to stop, and a perfect maintenance action to restore the machine to as good as the new state with a duration (t_p) is applied until the end of the production period. Then the new preventive maintenance plans according to cumulative failure rate are re-engineered from the end of this period, as indicated in Figure 2 above. The ultimate objective is the Sustainability of the production system through reliability improvement and the satisfaction of varied customers with various products at different quality standards while ensuring profitability in the present-day competition and dynamic global market forces. We optimize the total cost of production, which is the sum cost of all products (P1 & P2) production cost, the total inventory holding costs at all the principal stores, the multi-warehouses, the delivery costs, and the costs associated with the maintenance strategy, sampling inspection, and non-conformal products lost cost to carry out this collaborative production, delivery, quality and maintenance optimization.

3. Methodology

Firstly, we determine the optimal plan of production to satisfy the varied demand of different customers under a given service and respective quality level. Secondly, according to the economic goal of production, the optimal parameters of the control chart are characterized by the sample size n, the control interval h, the control limits coefficient A, and the optimal maintenance strategy associated with the optimal number of preventive maintenance actions N^*. The horizon is proportionally divided into H periods with a length equal to $\Delta\mathrm{t}$. Moreover, we assume that demand fluctuation is normal with mean $\mu_{,j}(k)$, and variance $V_{di,j}(k)$, which are known, respectively. The demands are satisfied at the end of each period.

3.1 Assumptions

Based on some industrial reality and limitations, the following hypotheses are considered. Repair and overhaul times are not necessarily considered insignificant in the context of maintenance tasks in the industrial setting. For this study, we constructed each proposed PMMR maintenance activity so that no time is required in the control state. The plan only intends to give us a general idea of the tools, resources, and technicians needed to plan the repair and shorten the time we intervene. This forecasting maintenance planning is distinguished by the ideal number of preventive maintenance actions and the gaps between two successive preventive maintenance. On the other hand, we consider duration (t_p) for each perfect maintenance corresponding to each out-of-control situation due to assignable causes. We consider the following hypotheses:

1. The random demand follows the normal law, and every unsatisfied request causes a delayed penalty.
2. The delivery time τ_i from S to each warehouse w_i is constant for all products (P1 & P2) and is multiple of Δt.
3. The products production bounds ($u^1{}_{min}$, $u^1{}_{max}$) and ($u^2{}_{min}$, $u^2{}_{max}$) are known and constant.

4. The repair and preventive activity duration are negligible when the production process is under control.
5. The times of perfect maintenance performed for each failure caused by the quality indicator, which happened to be located out of control, is t_p*, and a new maintenance plan is re-launch based on the cumulative failure model.
6. The non-conformal products need to be recovered.
7. Defective products are due to the degradation of the machine.
8. all unit costs are constant and known.
9. All resources to carry out maintenance activities are still available.

3.2 The Control Chart

The probability of missing an adjustment while taking a n sample size can be used by the control chart to gauge how effective a control card performs. The card's significance is inversely correlated with the probability, suggesting that it is advantageous when the probability is low and disadvantageous when it is high. The average can change and be taken as value when the process is faulty. We observe that the mean adjustment is represented in a number of standard deviations:

$$\delta_{i,j} = \frac{\mu_i - \mu_{i,0}}{\sigma_{i,0}} \quad \text{where i = Products = } \{1, 2\} \tag{5}$$

Let $P_{i,j}(c)$ be the probability of not detecting an adjustment of c standard deviations when taking a sample of c pieces.

$$P_{i,j}(\delta_{i,j}) = F(-\delta_{i,j}.\sqrt{n_i} + A_i) - F(-\delta_j.\sqrt{n_i} - A_i), \quad j = \{1, 2, \dots, \}, \text{ i} = \{1, 2\} \tag{6}$$

With F: distribution function of the reduced centered normal law.
Hence, the average operational period (AOP) characterizes the average number of successive samples leading to the first point out of range for a given adjustment and on any given product I, i = $\{1, 2\}$.

$$AOP_{i,j} = \frac{1}{1 - P_{i,j}(\delta_{i,j})} = \frac{1}{1 - F(-\delta_{i,j}.\sqrt{n_i} + A_i) - F(-\delta_{i,j}.\sqrt{n_i} - A_i)} \tag{7}$$

3.3 Production, inventory, and delay penalty costs formulation

Our goal is to simultaneously establish the optimum production rates, the optimal inventory quantities based on the demand, and the quality control chart's optimal parameters, which minimizes the total cost. The total expected cost includes the production cost, the inventory cost, the delay penalties, the quality cost (sampling inspection, false alarm & non-conformal products), and maintenance cost (preventive, minimal repairs, and corrective maintenance).

Production Cost

The production cost at period k is equal to the sum of production with respect to all products.

$$PC_T = \sum_{k=1}^{H} \{C_{p_1} \times u_1(k) + C_{p_2} \times u_2(k)\}.\Delta t \tag{8}$$

Holding cost at the primary production Store (S)

Each unit product put in stock anywhere and at any time will generate a unit cost of holding it, therefore, to minimize the total cost, the need to carefully plan for an optimal stock quantity according to the inventory management system to satisfy customer demand at some given service requirements. The inventory level balance equation characterizes the progress of the principle inventory for periods $(k = 1, 2 \dots, H)$, which is defined by:

$$S(k) = S(k-1) + (u_1(k) + u_2(k)).\Delta t - \sum_{i=1}^{L} Q_{1,i}(k - \tau_i) + Q_{2,i}(k - \tau_i), \text{ with } k = \{0, 1 \dots, H\} \tag{9}$$

$Z_S(k)$ is the area generated by the evolution of the inventory level at period k, $(k = 1, 2, \dots, H)$

$$Z_S(k) = Max\{S(k-1), 0\}.\Delta t + \frac{1}{2}.(u_1(k) + u_2(k)).\Delta t^2 \tag{10}$$

Consequently, the holding cost is expressed by:

$$HC_S = \sum_{k=1}^{H} C_h \times Z_S(k) \tag{11}$$

Where c_h = The unit cost of holding inventory.

Holding cost at the retail warehouses (w_i)

The following expression gives the inventory holding cost for the product at all the warehouses:

$$HC_W = \sum_{k=1}^{H} C_h \times Z_{w_i}(k) \quad (12)$$

$Z_{w_i}(k)$ is the area generated by the inventory level evolution during the period k, $(k = 1, 2, \dots, H)$

The following equation gives the inventory level of each warehouse inventory w_i at the period k:

$$w_i(k) = w_i(k-1) + Q_{1,i}(k-\tau_i) + Q_{2,i}(k-\tau_i) - d_{1,i}(k) - d_{2,i}(k) \quad (13)$$

The inventory level of each product in any warehouse w_i at any given period k equals the inventory level of w_i at period $(k-1)$ plus the quantity of the sum of all products that arrives *at* w_i (i.e., $Q_{1,i}(k-\tau_i)$ $+Q_{2,i}(k-\tau_i)$) minus all the customer demands $d_{1,i}$ & $d_{2,}$ I at period k.

The generated area of the inventory level evolution for each warehouse (w_i) during any given period k is given as follows:

$$Z_{w_i}(k) = Max\{w_i(k-1), 0\}.\Delta t + \frac{1}{2}(Q_{1,i}(k-\tau_i) + Q_{2,i}(k-\tau_i)).\Delta^2 \quad (14)$$

Consequently,

Total Inventory Holding cost (HC)

This consists of the inventory holding cost at the main production store (S), and all the costs of inventories from the multiple retail warehouses (w_i {i = 1, ... L}) expressed as follows:

$$HC_T = C_h \times \sum_{k=1}^{H} \begin{pmatrix} Max\{S(k-1), 0\}.\Delta t + \frac{1}{2}.\left(u_1(k) + u_2(k)\right).\Delta t^2 & + \\ Max\{w_i(k-1), 0\}.\Delta t + \frac{1}{2}(Q_{1,i}(k-\tau_i) + Q_{2,i}(k-\tau_i)).\Delta^2 & \end{pmatrix} \quad (15)$$

The service level

The following constraint expresses the service level requirement constraint for each warehouse at each period k. For (k= 1, …, H-1) and (i=1, …, L)

$$\text{Pro}\ [w_i(k) \geq 0] \geq \theta_i \quad (16)$$

Production bounds

The following constraint defines an upper and lower bound on the products' production levels during each period k.

$$u^1{}_{min} \leq u_1(k) \leq u^1{}_{max} \quad (17)$$

Also

$$u^2{}_{min} \leq u_2(k) \leq u^2{}_{max} \quad (18)$$

Total Production Cost

The production cost of all the products and for any period of k

$$PC_T = \sum_{k=1}^{H}\{C_{p_1} \times u_1(k) + C_{p_2} \times u_2(k)\}.\Delta t \quad (19)$$

Delay Penalties Cost

The delay penalties are characterized by the consequence of a delay in satisfying all the demands. If a delay situation occurred at the end of the period, k, causing a shortage recovered during the next period (k+1):

The penalties are determined as a function of the required duration dw (.) to produce the missed quantity at the end of each period, given by the following expression:

$$PC_T = (C_{p_1} + C_{p_2}) \times (\sum_{k=1}^{H} (\sum_{i=1}^{L} dw_i)), \quad dw_i = \frac{|\min(w_i(k)i,0)|}{Q(k+1-\tau_i)} \tag{20}$$

Maintenance Policy

Maintenance cost consists of the false preventive and corrective maintenance costs and also depends on the scenario "scenario 1: in-control" and "scenario 2: out-control" that happens. The resolution of the maintenance planning problem consists of minimizing costs related to preventive and corrective maintenance. The maintenance strategy considered in this work is preventive maintenance with minimal repair in the 'in-control' state of the system. Preventive actions are scheduled over the finite horizon H which is divided equally into N parts of duration T. We suppose that performing a preventive action corresponds to times i.T (I =1, 2, N) consists of restoring the machine to an as good as new condition. However, breakdowns may happen between successive preventive interventions, and minimal repair is performed. It is assumed that the repair and overhaul durations are negligible. Essentially the status of the machine depends on its average failure rate, influenced by the total production rates of the multi-products and delivery plan variation. Whereas the maintenance strategy is characterized by the optimal number of preventive maintenance actions N* and the most adequate spacing between them noted T*.

$$CM_T = cm_1 \times \left(P(S_{1,1}) + P(S_{1,2})\right) + cm_2 \times \left(P(S_{1,1}) + P(S_{1,2})\right) \tag{21}$$

$$P(S_{1,1}) = Prob(LCL^1 \geq \bar{X}_t \geq UCL^1) = F\left(\mu_{1,0} + \frac{A_1}{\sqrt{n_1}} \times \sigma_1\right) - F\left(\mu_{1,0} + \frac{A_1}{\sqrt{n_1}} \times \sigma_1\right) \tag{22}$$

$$P(S_{1,2}) = Prob(LCL^2 \geq \bar{X}_t \geq UCL^2) = F\left(\mu_{2,0} + \frac{A_2}{\sqrt{n_2}} \times \sigma_2\right) - F\left(\mu_{2,0} + \frac{A_2}{\sqrt{n_2}} \times \sigma_2\right) \tag{23}$$

$$P(S_{2,1}) = Prob(\bar{X}_t \leq LCL^1) + Prob(UCL^1 \leq \bar{X}_t) = F\left(\mu_{1,0} - \frac{A_1}{\sqrt{n_1}} \times \sigma_1\right) + 1 - F\left(\mu_{1,0} + \frac{A_1}{\sqrt{n_1}} \times \sigma_1\right) \tag{24}$$

$$P(S_{2,2}) = Prob(\bar{X}_t \leq LCL^2) + Prob(UCL^2 \leq \bar{X}_t) = F\left(\mu_{2,0} - \frac{A_2}{\sqrt{n_2}} \times \sigma_2\right) + 1 - F\left(\mu_{2,0} + \frac{A_2}{\sqrt{n_2}} \times \sigma_2\right) \tag{25}$$

$$cm_1 = C_{pm} \times \left\lfloor \frac{H}{T} \right\rfloor + C_{cm} \times \varphi_M(U, N) \tag{26}$$

Where $\varphi_M(U,N)$ is the average number of failures as a function of the production plan defined by the vector U and the number of preventive maintenance actions N^*, note also $\frac{H}{T} = N$.

$$\varphi_M(U,N) = \sum_{j=0}^{N-1} \left[\sum_{k=In\left(j\times\frac{N}{\Delta t}\right)+1}^{In\left((j+1)\times\frac{N}{\Delta t}\right)\times\Delta t} \int_0^{\Delta t} \lambda_k(t)dt + \int_1^{(j+1)\times N - In\left((j+1)\times\frac{N}{\Delta t}\right)\times\Delta t} \lambda_{In\left(j+1)\times\frac{N}{\Delta t}\right)+1}(t) + \int_{(j+1)\times N}^{\left(In\left((j+1)\times\frac{N}{\Delta t}\right)+1\right)\times\Delta t} \frac{\left(\left(In\left(j+1)\times\frac{N}{\Delta t}\right)+1\right)\right)}{\left(\frac{u_1(k)}{u^1{}_{max}} + \frac{u_2(k)}{{}^2{}_{max}}\right)} \times \lambda_n(t)dt \right] \tag{27}$$

With $\lambda(t)$ represents the linear failure rate function at production period k expressed as follows:

$$\lambda_k(t+1) = \lambda_k(\Delta t) + \left(\frac{u_1(k)}{u^1{}_{max}} + \frac{u_2(k)}{{}^2{}_{max}}\right).\lambda_n(t)dt \qquad \forall t \in [0, \Delta t] \tag{28}$$

$\lambda_n(t)$: failure rate for nominal conditions (maximal production during all horizon H.Δt).

$$cm_2 = \sum_{j=0}^{M} \left(C_{pm} \times \left\lfloor \frac{AOP_{j+1} - AOP_j}{N_j} \right\rfloor + C_{cm} \times \varphi_{Mj}(U, N_j) \right) + C_{pm} \times \left\lfloor \frac{H - AOP_M}{N_j} \right\rfloor + C_{cm} \times \varphi_M(U, N_M) \tag{29}$$

M: Number of detection of the out of control

$$\varphi_{Mj}(U,N_j)=\sum_{z=0}^{N_j}\sum_{k=\left\lfloor\frac{AOP_j}{N_j}\right\rfloor+\left\lfloor z\times\frac{N_j}{\Delta t}\right\rfloor+1}^{\left\lfloor\frac{AOP_j}{N_j}\right\rfloor+\left\lfloor(z+1)\times\frac{N_j}{\Delta t}\right\rfloor}\int_0^{\Delta t}\lambda_k(t)dt,\quad N_j=\left\lfloor\frac{AOP_{j+1}-AOP_j}{N_j}\right\rfloor \tag{30}$$

And

$$\varphi_{M}(U,N_M)=\sum_{z=0}^{N_M}\sum_{k=\left\lfloor\frac{AOP_M}{N_M}\right\rfloor+\left\lfloor z\times\frac{N_M}{\Delta t}\right\rfloor+1}^{\left\lfloor\frac{AOP_M}{N_M}\right\rfloor+\left\lfloor(z+1)\times\frac{N_M}{\Delta t}\right\rfloor}\int_0^{\Delta t}\lambda_k(t)dt\ ,\quad N_M=\left\lfloor\frac{H-AOP_M}{N_M}\right\rfloor \tag{31}$$

The average total quality cost

The average total cost of quality corresponds to the sum of the sampling inspection cost and the cost of rejection of non-conforming Products.

The expression for the average total cost of sampling is:

$$C_{Si}=\sum_{j=0}^{M}\Big(\big(C_{1,i}\times n_{1,i}\times(AOP_{1,(j+1)}-AOP_{1,j})\big)+\big(\big(C_{2,i}\times n_{2,i}\times(AOP_{2,(j+1)}-AOP_{2,j})\big)\big)\Big) \tag{32}$$

While the average total cost of rejection (c_{NCr}) products corresponds to the non-conforming production quantity during the out-of-control period expressed by:

$$c_{NCr}=\sum_{j=0}^{M}\left(\left(C_{1,r}\times u_1\times\left(\left\lfloor\frac{AOP_{1,j}}{\Delta t}\right\rfloor\right)\times\left(\frac{AOP_{1,j}}{\Delta t}-h\right)\right)+\left(C_{2,r}\times u_2\times\left(\left\lfloor\frac{AOP_{2,j}}{\Delta t}\right\rfloor\right)\times\left(\frac{AOP_{2,j}}{\Delta t}-h\right)\right)\right) \tag{33}$$

Subsequently, the average total cost of quality ACQ is given by:

$$ACQ=\sum_{j=0}^{M}\Big(\big(C_{1,i}\times n_{1,i}\times(AOP_{1,(j+1)}-AOP_{1,j})\big)+\big(\big(C_{2,i}\times n_{2,i}\times(AOP_{2,(j+1)}-AOP_{2,j})\big)\big)+\Big(C_{1,r}\times u_1\times\left(\left\lfloor\frac{AOP_{1,j}}{\Delta t}\right\rfloor\right)\times\left(\frac{AOP_{1,j}}{\Delta t}-h\right)\Big)+\left(C_{2,r}\times u_2\times\left(\left\lfloor\frac{AOP_{2,j}}{\Delta t}\right\rfloor\right)\times\left(\frac{AOP_{2,j}}{\Delta t}-h\right)\right)\Big) \tag{34}$$

Where; $AOP_{1,0}=0,\ AOP_{2,0}=0.$

4. Numerical Experiment

The data used in this analysis are established primarily based on a typical dynamic behavior for a forecasted two-product demand problem over one year.

4.1 The Numerical Example

Consider a supply chain management system composed of a production system that produces two types of products linked to two warehouses (*L*=2) to satisfy randomly varied demands over a finite planning horizon: *H* =12 months each of period length Δt=1 month. The standard deviation of each product is identical for all periods, and for each demand $\sigma_{di(\{i:1,2\})}=20$, and the initial inventory level, we assume that $S(0)=0$. The average demand for product 1, customers of warehouse 1 and warehouse 2: $d_{1,1}(k)=d_{1,2}(k)=300$, while the average demand for product 2, customers of warehouse 1 and warehouse 2: $d_{2,1}(k)=d_{2,2}(k)=30$, $\{k: 0, \ldots, H-1\}$.

- Lower and upper boundaries of product 1 production capacities: $u^1{}_{min}=0\ and\ u^1{}_{max}=500$
- Lower and upper product 2 production capacities boundaries: $u^2{}_{min}=0\ and\ u^2{}_{max}=50.$
- $C_{p,1}=2$ *mu*, $C_{p,2}=5$ *mu*, $S_{i,1}(0)=0$, $S_{i,2}(0)=0$, $C_{hs}=0.2$ *mu/k*,
- $w_{i,1}(0)=10$, $w_{i,2}(0)=10$, $Cw_i=0.2$ *mu/k* {i:1,….*L*=2}.
- $\mu_{0,1}=5$, $\sigma_{0,1}=1.5$, $\mu_{0,2}=3$, $\sigma_{0,2}=1.2$ and $\delta_1=0.8$, $\delta_2=0.4$
- $Ci=15$ *mu* /product, $Cr=70$ *mu* /defective item.
- customers' satisfaction degree equals 90% (θ_i=0.9 (*i*=1, 2)).
- The degradation law of the production system characterized by a Weibull distribution with scale and shape parameters are, respectively, $\beta_1=100$ and $\alpha_1=2$.
- The costs of corrective and preventive maintenance actions are respectively Ccm =3000 *mu* and Cpm=500 *mu*

The characterized varied demand, optimal production of products (P1 & P2), and delivery plans for each warehouse 1 and 2 according to $(Q_1, I, Q_{2,i})$ are given in Table I and II, respectively:

Table 1: Customers random demand

Period (k)	$d_{1,1}$	$d_{1,2}$	$d_{2,1}$	$d_{2,2}$
1	300	320	30	32
2	340	260	34	26
3	300	300	30	30
4	300	300	30	31
5	300	280	31	27
6	280	320	28	32
7	320	320	32	32
8	280	320	29	30
9	320	280	31	28
10	300	300	31	29
11	300	300	32	30
12	300	280	33	28

Table 2: Optimal production and delivery Quantities plan.

K	$u_{1,1}$	$u_{1,2}$	$u_{2,1}$	$u_{2,2}$	$Q_{1,1}$	$Q_{1,2}$	$Q_{2,1}$	$Q_{2,2}$
1	492	492	20	30	217	133	33	21
2	488	488	26	29	393	302	20	23
3	406	406	29	24	250	356	25	33
4	465	465	26	28	220	260	20	19
5	385	385	18	22	328	278	11	27
6	420	490	30	30	337	354	21	23
7	489	489	29	28	260	244	19	21
8	260	260	28	25	332	239	24	21
9	459	498	25	25	216	251	23	32
10	244	244	23	26	231	270	21	12
11	334	334	17	27	356	230	22	24
12	250	350	11	31	340	401	25	21

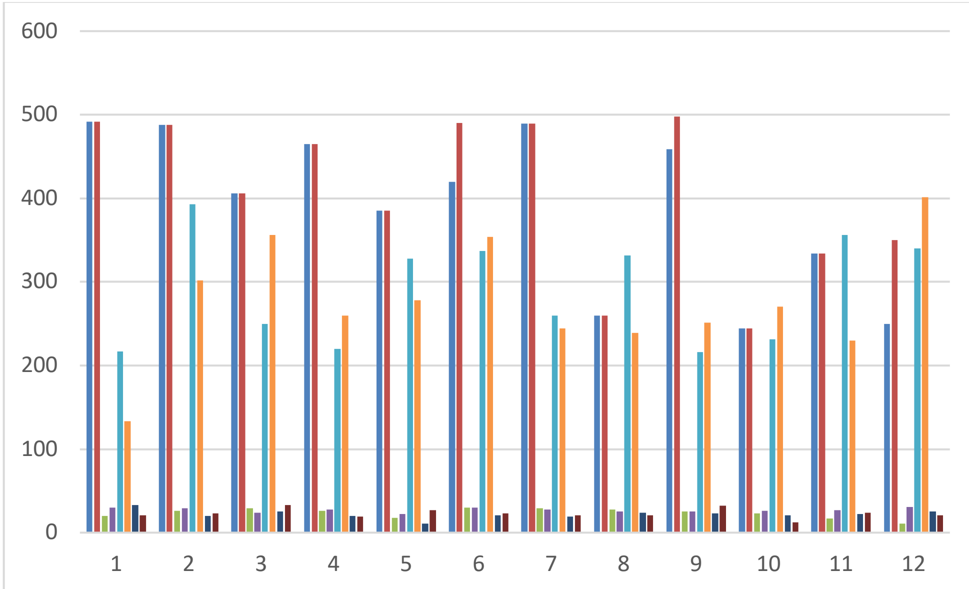

Figure 3: The evolution of 2 products' production and inventory plans

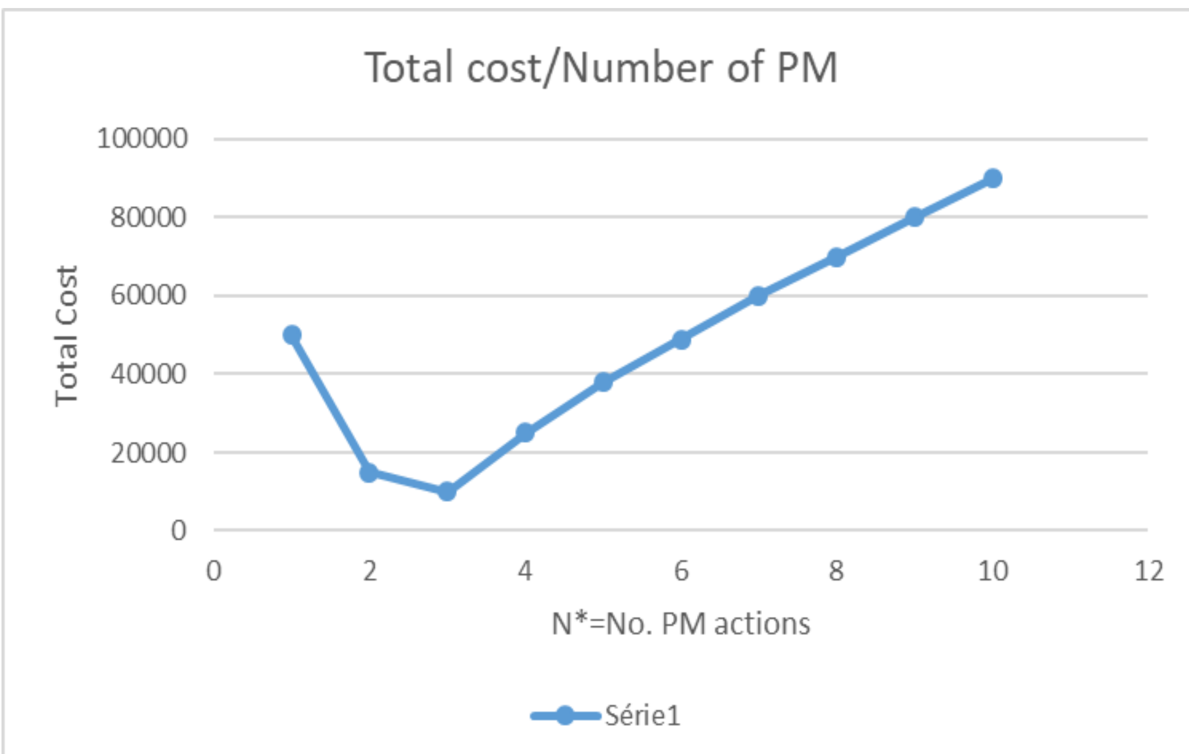

Figure 4: Optimal Number of Maintenance Action

4.2 The Analysis of Results and Discussion

The obtained strategy consists of taking one sample size ($n_1 = 60$) of product 1 every 96 hours = 4 days ($h*$). Furthermore, according to the control chart parameters associated with product 1, the optimal number of standard deviations between the center line and the control limits is 3.5 (A_1*). similarly for product 2 by taking one sample size ($n_2 = 14$) of product 2 every 96 hours = 4 days ($h*$). Furthermore, according to the control chart parameters associated with product 2, the optimal number of standard deviations between the center line and the control limits is 2.5 (A*). The first shift of product 1 to the first 'out of control' state occurs on average operational period AOP1 = 4. Δt = 24 samples, while that of Product 2 happens much later due to its wide tolerance limits. The cumulative effects of the total production strategy on the single machine, as indicated in Figure 3 above, requires that 3PM actions are carried out every T = 4. Δt = 16. h and 3T = 12.Δt = 48. h.

5. Conclusion

This paper studied an integrated approach to a single-machine multi-product production system's production, maintenance, and quality problem to satisfy random demands, as presented in Table 1. We consider a cumulative failure due to the impacts of varied production rates and the inventory use of the multi-product case on the model formulation. We developed a mathematical model which determines the optimal number of PM actions that minimize the total cost of production according to the different decision variables. As presented in Table 2, we achieved a collaborative optimization of the production plan, inventory management, and maintenance strategy integrated into the quality control. The numerical experiment indicated the proposed model's robustness and capabilities for economic gains to production companies, especially the multi-product production set-ups. The study's main finding showed the strong interrelation of the three essential production functions. The integrated model led to a significant increase in process reliability and a reduction of losses (NC-products). Adequate coordination of these factors provided by the control chart minimizes the total production costs.

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